

Mean & Variance

⇒ Discrete Random Variable

The complete description of a discrete random variable requires specifying the possible values that the random variable may take and the probability associated with each value (probability distribution)

X	1	2	3	4	...
P(X)	✓	✓	✓	✓	...

⇒ Mean: (expectation, expected value)

$$E(X), \mu_x, \mu$$

$$\text{First moment of the origin} = \sum_x X P(X)$$

$$\text{Second moment of the origin} = \sum_x X^2 P(X)$$

$$r^{\text{th}} \quad // \quad // \quad // \quad // \quad = \sum_x X^r P(X)$$

⇒ Variance

$$\begin{aligned} \text{1st moment about the mean} &= E(X - \mu) \\ &= \sum_x (X - \mu) P(X) \end{aligned}$$

$$\begin{aligned} \text{2nd moment about the mean} &= E(X - \mu)^2 \\ &= \sum_x (X - \mu)^2 P(X) \\ &= \text{Variance} \\ &= \text{Var}(X) = \sigma^2 \end{aligned}$$

⇒ Standard Deviation: σ (S.D.)

$$\sigma = \sqrt{\text{Var}(X)}$$

Ex: Prove that $\text{Var}(X) = E(X^2) - \mu^2$

$$\begin{aligned} \text{Var}(X) &= \sum_x (X - \mu)^2 P(X) \\ &= \sum_x (X^2 - 2\mu X + \mu^2) P(X) \\ &= \sum_x X^2 P(X) - 2\mu \sum_x X P(X) + \mu^2 \sum_x P(X) \\ &= E(X^2) - 2\mu^2 + \mu^2 \quad \begin{matrix} \text{E} \\ \text{E} \end{matrix} \\ &= E(X^2) - \mu^2 \end{aligned}$$

⇒ Continuous Random Variable

⇒ Mean

$$\text{1st moment about the origin} = E(X) = \int_{-\infty}^{\infty} x P(x)$$

$$\text{2nd} \quad // \quad // \quad // \quad // \quad = E(X^2) = \int_{-\infty}^{\infty} x^2 P(x)$$

⇒ Variance

$$\text{1st moment about the mean} = E(X - \mu) = \int_{-\infty}^{\infty} (X - \mu) P(x)$$

$$\begin{aligned} \text{2nd} \quad // \quad // \quad // \quad // \quad &= E(X - \mu)^2 = \int_{-\infty}^{\infty} (X - \mu)^2 P(x) \\ &= \text{Variance} \end{aligned}$$

Sheet 4

7 Plot the graph of $F(x)$ for the variable X with density

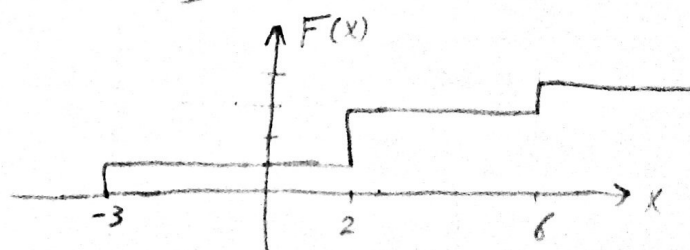
X	-3	2	6
P(X)	0.25	0.5	0.25

$$F(-\infty) = 0$$

$$F(-3) = P(-3) = \frac{1}{4}$$

$$F(2) = P(2) + P(-3) = \frac{3}{4}$$

$$F(6) = 1$$



Sheet 5

- ① X equal the no. of Successful Cures out of the Five

X	0	1	2	3	4	5
P(X)	0.002	0.029	0.132	0.309	0.36	0.168

Find μ, σ

$$\begin{aligned}\mu &= \sum_x x P(x) = 0(0.002) + 1(0.029) \\ &\quad + 2(0.132) + 3(0.309) + 4(0.36) \\ &\quad + 5(0.168) \\ &= 3.5\end{aligned}$$

$$\begin{aligned}\sigma^2 &= E(x^2) - \mu^2 = \sum_x x^2 P(x) - \mu^2 \\ &= 0(0.002) + 1(0.029) + 2^2(0.132) \\ &\quad + 3^2(0.309) + 4^2(0.36) + 5^2(0.168) \\ &\quad - (3.5)^2 \\ &= (1.02)^2\end{aligned}$$

$$\sigma = 1.02$$

- ② iv) Find the expectation, Variance, S.D.

$$P(x) = \begin{cases} \frac{2}{25} x & 0 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned}\mu &= \int_{-\infty}^{\infty} x P(x) dx = \int_0^5 \frac{2}{25} x^2 dx \\ &= \frac{2}{25} \left[\frac{x^3}{3} \right]_0^5 = \frac{10}{3}\end{aligned}$$

$$\begin{aligned}\text{Var}(X) &= E(x^2) - \mu^2 \\ &= \int_{-\infty}^{\infty} x^2 P(x) dx - \mu^2 \\ &= \checkmark\end{aligned}$$

$$\sigma = \sqrt{\text{Var}(X)} = \checkmark$$